Mock Exam Answers

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Part 1, Question 1

i Unclear, the outcome (B_1, A_2) can certainly arise in the mixed strategy NE; on the other hand, it is not clear what occurring as a strategy profile means.

ii False, counter example:

	L	R
U	3	0
M	0	3
D	1	1

iii False, consider a trivial game where all payoffs are the same

iv False, consider any pure strategy profile (so degenerately a mixed strategy) that is not NE.

Or consider more obvious case where the support of the mixed strategy excludes a strictly dominant strategy

Part 1, Question 2

a Best responses of 1 to y: $[0, 1-y]$. Best responses of 2 to x: $[0, 1-x]$. Best response graphs entirely overlap; form triangle with vertices $(0,0), (0,1), (1,0)$.

b Any point in this triangle is NE; i.e. any (x, y) with $x + y \leq 1$

c Pareto efficient on frontier: (x, y) with $x + y = 1$

d Giving 0 is the only weakly dominant strategy. So $(0, 0)$

e Yes. Strategy for 2 now function $y(x)$. Suppose $y(x) = 0$ for all x. Weakly dominant for 2. Any x is a best response for 1.

f Yes, this is sub-game perfect. Given x, 2's best responses are $[0, 1-x]$, which always include 0. Player 1 is indifferent between all strategies and so trivially is best responding.

Part 1, Question 3

a (c, c), $p^m = (a + c)/2$

b i Assume $p \geq c$, otherwise no SPE.

Consider Nash reversion strategy profile. Need to check that there are no profitable one stage deviations. Off-path, this is true as the threat point is NE. On path, need:

$$\frac{\Pi(p)}{n} \frac{1}{1 - \delta} \geq \Pi(p) + 0 \iff \delta \geq 1 - \frac{1}{n}$$

b ii By Folk theorem, yes. Best deviation is now to p^m, so the above condition becomes

$$\frac{\Pi(p)}{n} \frac{1}{1 - \delta} \geq \Pi(p^m) + 0 \iff \delta \geq 1 - \frac{1}{n} \frac{\Pi(p)}{\Pi(p^m)}$$

Check: higher monopoly profits, greater incentive to deviate, so requires more patience.

Part 1, Question 4

a Backward induction. Play static NE at terminal histories. $\cdots$ Play at any history does not alter future play, so play static NE there, too. No.

b i Posit SPE: play (A, A) in period 1, Pareto dominant NE (C, B) in period 2, if deviation play Pareto inferior NE (B, C). There is no incentive to deviate in period 2 since play is static Nash. In period 1, we require $6 + a \geq 7 + b$, which holds since $a > b + 1$, and $6 + b \geq 7 + 1$, which holds since $b > 3$.

Posit SPE: same as above except now play (A, B) in period 1. Again, there is no incentive to deviate in period 2 since play is static Nash. In period 1, we require $7 + a \geq 7 + b$, which holds since $a > b$, and $0 + b \geq 1 + 1$, which holds since $b > 3$.

Posit another SPE: same as above except now play (B, A) in period 1. This is not an SPE since 1 has an incentive to deviate in period 1: $0 + a < a + b$

b ii There are no pure NE. The minmax point is (1,1). By Folk theorem, yes as average of per-period payoffs strictly dominate minmax point

Part 1, Question 5

a A mapping from the set of all profiles of rational preferences over a given number of alternatives to the set of rational preferences over those alternatives; the mapping is Pareto efficient and satisfies independence of irrelevant alternatives: if two profiles rank two alternatives in the same way, then their images rank these alternatives in the same way.

b Only Arrow social choice rule is dictatorship. Let's make 3 the dictator.

c 3 dictator. Use her ranking.

d i Pareto efficiency

d ii Condorcet's paradox: no alternative that can beat every other. Possible resolution via secondary round, e.g. run-off vote.

Part 2, Question 3

a Tie compensation to outcome: bonus for g, punishment for b. Unobservable trading choice induces tradeoff between aligning incentives and income insurance, assuming the CEO is more risk averse than the government.

b i Pay a constant wage as the CEO is risk averse and the government is risk neutral: $w_z(e) = w(e)$. CEO expected utility is $\max(u(w(a))+d_a, u(w(p))+d_p)$. If the CEO prefers a, then the government's expected payoff is $\mathbf{E}_\pi[z|a] - w(a)$. If the CEO prefers p, then the government's expected payoff is $\mathbf{E}_\pi[z|p] - w(p)$.

b ii To implement p use $w(a) = 0$,

$$w(p) = \max(u^{-1}(\bar{u} - d_p), u^{-1}(u(0) + d_a - d_p)),$$

the larger of the wage necessary to achieve reservation utility and the wage necessary to induce the CEO to choose p. Likewise, to implement a use $w(p) = 0$ and

$$w(a) = \max(u^{-1}(\bar{u} - d_a), u^{-1}(u(0) + d_p - d_a)).$$

Action yielding higher expected payoff for government implemented.

b iii Government bears all risk. CEO action observable implies no tension between incentives and insurance.

Part 2, Question 4

a All, quality q sells at price between q and $(3/2)q$. Yes.

b 3/4. No, sellers $S > 3/4$ will not

c No cars sold. No. Market unravels. Want to make car quality observable to buyers, could do e.g. via third-party certification

Part 2, Question 5

a h imitates l, receiving same wage for less hours worked: $w - t_h/h < w - t_l/h$.

b i $\pi(h\sqrt{t_h} - w_h) + (1-\pi)(l\sqrt{t_l} - w_l)$.

b ii Participation constraint $w_h - t_h/h \geq 0$ and incentive constraint $w_h - t_h/h \geq w_l - t_l/h$. For l: $w_l - t_l/l \geq 0$ and $w_l - t_l/l \geq w_h - t_h/l$.

c i Incentive constraints no longer relevant. Need only participation constraints

c ii $w_h = t_h/h$. Maximise $h\sqrt{t_h} - w_h$. Gives $t_h = h^4/4$, $w_h = h^3/4$