Problem Set 3

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Problem 1

A player's strategy specifies what action to take following each history where she can take an action. We multiply the number of actions at each of these histories. Player 1 has 4 strategies, player 2 has 3^4, player 3 has 2^{12}.

Problem 2

Player A's strategies are *both A*, *share*, and *both B*. A strategy for player B says whether to accept (a) or reject (r) following each possible proposal from A. We compute the subgame perfect equilibria using backward induction as follows. If A chooses *both A*, then B is indifferent between accepting and rejecting. If A chooses to share or to give both to B, then B's best response is to accept. Given B plays aaa, A's best response is *both A*. Given B plays raa, A's best response is *share*. So there are two subgame perfect equilibria: (*both A*, aaa), (*share*, raa).

The point of the question is to demonstrate that there are Nash equilibria that are not subgame-perfect since they involve incredible threats. For example, rra is not credible, but if B plays rra and A plays her best response *both B*, the resulting strategy profile is a Nash equilibrium whose outcome is that both items go to B—an outcome not arrived at by either subgame perfect equilibrium.

Problem 3

A strategy for firm 1 is a price $p \geq c$ and a strategy for firm 2 is a price $q(p) \geq c$ for each price p that 1 can select. To compute subgame perfect equilibria, use backward induction. If $p = c$, 2's best response is any price $\geq c$. If $p = c + 1$, then 2's best response is $c + 1$, too. Let p^M denote the monopoly price. If $p^M > p > c + 1$, then 2's best response is $p - 1$. Finally, if $p \geq p^M$, then 2's best response is p^M (or maybe $p^M - 1$, it will depend on the demand function and minimum unit of price). Consider any such strategy for firm 2. 1's best response is $p = c + 1$.

Problem 4

We solve for the subgame perfect equilibria using backward induction. Firm 3 sets the profit maximizing quantity $q_3^* = \arg\max_{q_3} p(q_1 + q_2 + q_3)q_3 = (1 - q_1 - q_2)/2$. Given this, 2's best response is $q_2^* = \arg\max_{q_2} p(q_1 + q_2 + q_3^*)q_2 = \cdots = (1 - q_1)/2$. And given this, 1's best response is $q_1^* = \arg\max_{q_1} p(q_1 + q_2^* + q_3^*)q_1 = \cdots = 1/2$. Hence $q_2^* = 1/4$ and $q_3^* = 1/8$. In general, with n firms, $q_n^* = 1/2^n$.

Problem 5

In the finitely repeated case we can see, using backward induction, that (U, L) will be the outcome in the last round, therefore it will be the outcome in the second-to-last round, and so on all the way back to the first round. In the infinitely repeated case, the folk theorem for repeated games with perfect information says that, for sufficiently patient players, all feasible payoffs that strictly dominate some static Nash equilibrium's payoffs can be supported as a subgame perfect equilibrium using a grim-trigger strategy with the static Nash equilibrium as eternal punishment. (The same applies if we replace Nash equilibrium payoffs with minmax payoffs, but then the punishment is only carried out for a limited amount of time). The one stage deviation property tells us that we only need to consider one-stage deviations. A player's continuation payoff from deviating is $5 + 2\delta/(1-\delta)$. A player's payoff from sticking to the Nash-reversion strategy is $4/(1-\delta)$. For the deviation not to be profitable we require $\delta \geq 1/3$.

(We have only checked that a one-stage deviation is not profitable *on the path of play*. We really should check other histories, those for which a defection has occurred in the past. Note, however, that this condition does not place any restriction on the discount factor since at such histories the strategy specifies play of a Nash equilibrium. On the other hand, if the punishment were to minmax for n periods, then this could impose a condition on the discount factor (and n).)

Problem 6

The monopoly price is $p^M = \arg\max_p (p-c)(a-p) = (a+c)/2$. Marginal cost pricing is the unique Nash equilibrium of the stage game. Repeating the analysis from Problem 5, again exploiting the one-stage deviation property, yields some threshold discount factor above which the Nash reversion strategy is a subgame perfect equilibrium. Adding firms reduces the monopoly profit of each firm and therefore reduces the set of discount factors for which this Nash-reversion strategy can be sustained as a subgame perfect equilibrium.